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Research article

Deterministic Model of Elastic Rough Contact Taking into Account the Mutual Influence of Asperities

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A B S T R A C T

The analytical solution of the elastic contact problem for the bodies with rough surfaces is presented using a deterministic description of the surface roughness. Particularly, a model of the normal contact of a periodic system of axisymmetric punches of different heights with an elastic half-space is developed. The solution is obtained taking into account the real contact pressure distribution only under nearby ones, and considering the action of remote ones as the nominal pressure (the localization method). The effect of the shape of the punches and their spatial positions on the dependencies of the real contact area and the contact pressure distribution at the contact spots on the applied load are studied. The additional displacement function, which characterizes the effect of surface roughness on the relative approach of contacting bodies at the macrolevel, is also analysed.

1. INTRODUCTION

Contact of real bodies never occurs over the entire apparent contact area as it is assumed, for example, in the Hertz theory [1]. The real contact area is a set of individual spots and so it is only a part of the apparent one. This fact is of great practical importance.

There are various approaches to solving contact problems taking into account the roughness of bodies, namely fractal, statistical and deterministic. The first approach assumes the fractal nature of the body surface and so uses the fractal geometry to describe the surface roughness and to develop the method of the contact problem analysis [2]. The second approach assumes that the rough surface profile can be described by some statistical distribution. The most famous such model is the Greenwood-Williamson model, which uses Hertz theory for asperities and takes their height distribution as normal [3]. These two approaches make it possible to obtain an effective solution of the contact problems, but in most cases, they neglect the mutual influence of the asperities.

With a deterministic approach, the height, shape and position of all asperities are known. In this case, the mutual influence can be included in the mechanical model straightaway. However, for real surfaces, it is very time-consuming due to the large amount of information. Therefore, with this approach, the use of numerical methods is common [4]. An effective numerical solution can be obtained using the method of reduction of dimensionality [5,6]. The task is noticeably simplified if the roughness has a periodic character [7], which makes it possible to get an analytical solution [8].

In this work, the normal contact of periodic system of punches with different heights and an elastic half-space is considered. Applying of the localization principle [8] allows us to study the influence of the shape of punches and the distance between them on the characteristics of the contact interaction and the real contact pressure distribution at the punches of the different heights.

2. STATEMENT OF THE PROBLEM AND METHOD OF ITS SOLUTION

The normal frictionless contact of the two-level periodic system of identical punches with the elastic half-space is considered. All punches have the same surface shape described by the function $f(r) = Cr^n / R^{n-1}$ (*R* characterizes the punch size, *C* is a dimensionless constant, *n* is a natural number). The height difference Δh of the punches in the system is given. At each level, punches are located in the nodes of a quadratic lattice with a given pitch *l*. The system is loaded by the nominal pressure $\bar{p}$. The nominal pressure is related to loads P_1 and P_2 (applied to a single punch of each level) as follows

$$l^2\bar{p} = P_1 + P_2. \quad (1)$$

To solve the periodic contact problem, the localization method is used [8]. According to this method, the contact pressure under an arbitrary punch can be obtained taking into account the real contact pressure distributions under nearby punches and replacing the action of others by the nominal pressure $\bar{p}$. So the following expression for the contact pressure $p_i(r)$ under the punch of the i^{th} ($i = 1,2$) level can be derived [9]

$$p_i(r) = p_0(r, a_i) + \frac{4}{\pi^2}\left(\frac{P_i\sqrt{a_i^2 - r^2}}{(l^2 - r^2)\sqrt{l^2 - a_i^2}} + \frac{P_j\sqrt{a_i^2 - r^2}}{(l_2^2 - r^2)\sqrt{l_2^2 - a_i^2}}\right) + \frac{2\bar{p}}{\pi}\arctan\left(\frac{\sqrt{a_i^2 - r^2}}{\sqrt{A_i^2 - a_i^2}}\right), \quad (2)$$

where $r \le a_i$, $i \ne j$, $l_2 = l/\sqrt{2}$, $p_0(r, a_i)$ is the pressure under the isolated punch of the given shape and the given contact radius a_i (expressions for various shapes described by the power function are given, for example, in [10]), and A_i is the radius of the region outside of which $\bar{p}$ acts. Eq.(2) is derived assuming the contact pressure under the nearest eight punches of both levels can be replaced by the forces $4P_1$ and $4P_2$ distributing on circles with radii *l* and l_2. The radii A_i ($i = 1,2$) are determined from the following equations

$$\pi A_i^2 \bar{p} = 5P_i + 4P_j. \quad (3)$$

The system of five equations (1)-(3) is added by two equilibrium equations and the equality of the difference in the displacements of the half-space surface under the centers of the punches of both levels and the given height difference Δh.

3. NUMERICAL STUDY OF CONTACT CHARACTERISTICS

The presented approach to solving the periodic contact problem allows us to determine the contact pressure distribution, the size of the contact area, and the redistribution of loads between punches of two levels. In this section, the dependences of the contact characteristics on the applied nominal pressure and the influence of the shape and location density of the punches on these characteristics are analyzed. In all numerical calculations, it was assumed that $C = 1$. The profiles of the three punch shapes in this case are shown in fig.1.

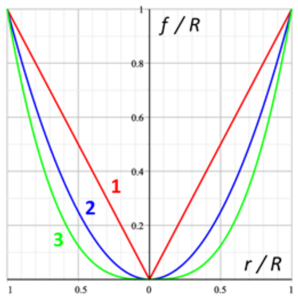


Fig. 1. Dimensionless profiles of punches of different shapes ($n=1$ (1), $n=2$ (2), and $n=3$ (3)).

3.1 Relative real contact area

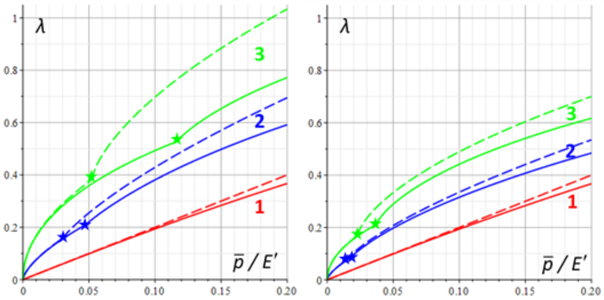
To study the real contact area, the following dimensionless function is introduced

$$\lambda = \frac{\pi}{l^2}(a_1^2 + a_2^2). \qquad (4)$$

This function characterizes the ratio between the real contact area and the nominal one.

Fig.2 presents dependences of the function λ on the applied dimensionless nominal pressure $\bar{p}(1-\nu^2)/E = \bar{p}/E'$ (E and ν are the Young's modulus and the Poisson's ratio of the half-space, respectively) for three punch shapes, namely for three values of the parameter *n*. For comparison, the results of calculations neglecting mutual influence of the punches are also presented in fig.2 by dashed lines. The inflection points on the graphs (star symbols in fig.2a and fig.2b) correspond to the beginning of the indentation of the second-level punches into the half-space. The results indicate that an increase in the parameter *n* leads to an increase in the real contact area. Additionally, neglecting the mutual influence leads to overestimation of the contact area size. This effect is more essential with decreasing of the lattice pitch (increasing of the contact density). A decrease in the density of the punches location in the system leads to a decrease in the relative contact area λ for all the shapes under consideration, with the exception of the conical one ($n=1$, curves 1 in fig.2a and fig.2b). Note that the difference in the results calculated with and without consideration of the punches interaction also decreases with a decrease in the density, i.e. in the lattice pitch. For example, at the nominal pressure $\bar{p} = 0.2E'$, increasing the lattice pitch *l* by one and a half times reduces the error from 17% to 11% for $n=2$ and from 34% to 14% for $n=3$.

In the case of conical punches, varying the density of the punches location practically does not change the relative contact area (compare the curves 1 in fig.2a and fig.2b). Moreover, if the mutual influence is not taken into account (dashed curves 1 in fig.2a and fig.2b) the following expression for the function λ is obtained.



(a) (b)

Fig. 2. Dependences of the relative contact area on the dimensionless nominal pressure for two lattice pitches ($l=R$ (a) and $l=1.5R$ (b)) and three punch shapes ($n=1$ (1), $n=2$ (2), and $n=3$ (3)) obtained with (solid lines) and without (dashed lines) the mutual influence; $\Delta h = 0.1R$.

$$\lambda = \frac{2}{l^2 c}(P_1 + P_2) = \frac{2\bar{p}l^2}{l^2 c} = 2\bar{p}. \qquad (5)$$

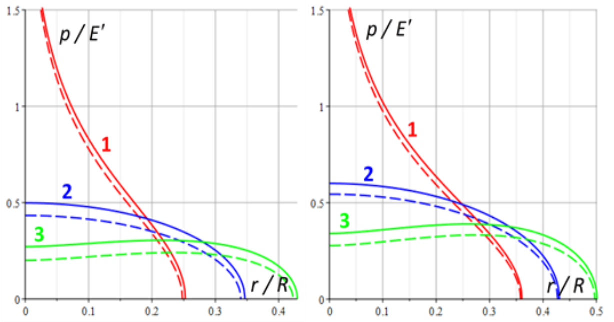
Thus, due to the linearity of the dependence of the contact spot area on the load, the relative contact area does not depend on the density of the punches location in the system. With a single-level contact, the absence of the influence of the lattice pitch on the dependence can also be proved analytically. Indeed, integrating (2) and using the equilibrium equation, we can derive the following expression determining the dependence of the relative contact area on the nominal pressure for the single-level contact

$$\frac{\bar{p}}{E'}\left(5\arccos\left(\sqrt{\frac{\lambda}{5}}\right) - 4\arccos\left(\sqrt{\frac{\lambda}{\pi}}\right) + \sqrt{\lambda(5-\lambda)} - \frac{4\lambda}{\sqrt{\pi-\lambda}}\right) = \frac{\pi}{4}C\lambda. \qquad (6)$$

3.2 Contact pressure distribution

Eqs.(2) can be used for calculation of the contact pressure distributions under the punches of both height levels. In this subsection the distributions under the punches of only the first height level are investigated, as more loaded areas.

Fig.3 demonstrates the dimensionless contact pressure distributions $p(r/R)/E'$ under the first level punches with three shapes of punches ($n=1,2,3$ for curves 1,2,3, respectively) and two various location densities (fig.3a and fig.3b). The results indicate that the maximum value of the contact pressure shifts to the edge of the contact spot as the parameter n increases. The influence of the nearby punches (increasing the location density) leads to an increase in the pressure in the central part of the contact spot in all cases. This increase is greater for the systems with a smaller lattice pitch l (the higher location density). For example, in the case of spherical punches ($n=2$), consideration of the mutual influence leads to a growth in the maximum pressure value $p_{\max}=p|_{r=0}$ by 15% for the system with $l=R$ and by 10% for the system with $l=1.5R$.



(a) (b)

Fig. 3. Distributions of the dimensionless contact pressure under the first level punch for two lattice pitches ($l=R$ (a) and $l=1.5R$ (b)) and three punch shapes ($n=1$ (1), $n=2$ (2), and $n=3$ (3)) obtained with (solid lines) and without (dashed lines) the mutual influence; $\Delta h=0.1R$, $\bar{p}=0.15E'$.

4. ADDITIONAL DISPLACEMENT FUNCTION

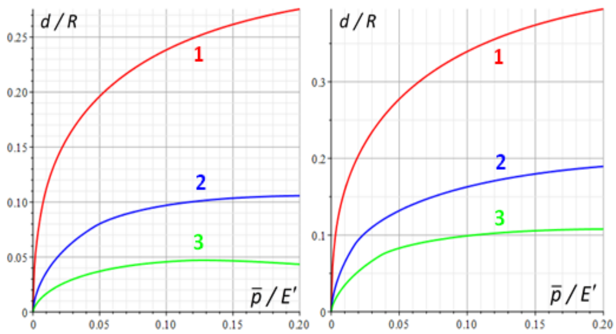
For taking into account the contacting bodies surface roughness in consideration of the contact problem at macroscale, the additional displacement (microgeometry "compliance") function must be calculated for the definite microgeometry parameters corresponding to measured surface roughness [11,12]. To study this characteristic, let us define the additional displacement function d, which is the difference between the vertical displacement $u_z^i(r)$ of the highest punch ($i=1$) in the central point and the displacement of the half-space boundary loaded everywhere with the nominal pressure. The following expression for d can be derived [9]

$$d = u_z^1(0) + \frac{4\bar{p}}{\pi E'}\int_0^{\infty} \mathbf{K}\left(\frac{2\sqrt{rr'}}{r+r'}\right)\frac{r'dr'}{r+r'} = d_0 + $$

$$+\frac{4}{\pi E'}\left(\frac{P_1}{\sqrt{l^2-a_1^2}}+\frac{P_2}{\sqrt{l_2^2-a_1^2}}\right)-\frac{2\bar{p}}{E'}\sqrt{A_1^2-a_1^2}, \quad (7)$$

where d_0 defines the indentation of an isolated punch into the elastic half-space for the contact radius a_1 [13], and $\mathbf{K}(x)$ is the complete elliptic integral of the first kind.

The dependences of the dimensionless additional displacement d/R on the dimensionless nominal pressure $\bar{p}/E'$ are presented in fig.4. As in the previous analysis, three shapes of the punches and two lattice pitches are considered. It follows from the results that a decrease in the parameter n and an increase in the lattice pitch l lead to a growth of the additional displacement. Moreover, in all considered cases, with an increase in the nominal pressure, the growth rate of the function $d(\bar{p})$ decreases. However, due to the replacement of the real contact pressure distribution under nearby punches with the load distributed on the circle, the displacement d begins to decrease at large values of $\bar{p}$ (line 3 in fig.4a). This demonstrates the limitations of the presented model. Thus, if the contact spots are very close to each other, the real contact pressure distributions at the nearest contact spots must be taken into account.



(a) (b)

Figure 4. Dependences of the dimensionless additional displacement on the dimensionless nominal pressure for two lattice pitches ($l=R$ (a) and $l=1.5R$ (b)) and three punch shapes ($n=1$ (1), $n=2$ (2), and $n=3$ (3)); $\Delta h=0.1R$.

The expression (7) can be used to formulate and solve the contact problem at the macrolevel and for analysis the influence of the microgeometry characteristics on the nominal and real contact pressure distribution in contact of the elastic bodies with given macroshape and microgeometry parameters.

5. CONCLUSION

In this work, the normal frictionless contact of a two-level periodic system of axisymmetric punches and an elastic half-space is investigated. The solution of the contact problem is obtained using the localization principle. The influence of the punches shape and their location density on the relative contact area and the contact pressure distribution is analyzed. It is shown that the geometric parameters of the system of punches also affect the magnitude of the error due to neglecting the mutual influence of contact spots.

Furthermore, the expression for the additional displacement due to surface microgeometry is also derived. The results indicate that the additional displacement significantly depends on the roughness parameters including the location density.

Note also that the contact pressures calculated above can be used to study the distribution of the internal stresses within the elastic half-space based on the analytical approach developed in [14].

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REFERENCES

[1] H. Hertz, "Über die Berührung fester elastischer Körper," *Journal für die reine und angewandte Mathematik*, vol. 1882, no. 92, pp. 156-171, 1882.

[2] A. Majumdar and B. Bhushan, "Fractal model of elastic-plastic contact between rough surfaces," *Journal of Tribology*, vol. 113, no. 1, pp. 1-11, 1991.

[3] J.A. Greenwood and J.B.P. Williamson, "Contact of nominally flat surfaces," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 295, no. 1442, pp. 300-319, 1966.

[4] V.A. Yastrebov, J. Durand, H. Proudhon, and G. Cailletaud, "Rough surface contact analysis by means of the finite element method and of a new reduced model," *Comptes Rendus Mécanique*, vol. 339, no. 7-8, pp. 473-490, 2011.

[5] R. Pohrt and V.L. Popov, "Investigation of the dry normal contact between fractal rough surfaces using the reduction method, comparison to 3D simulations," vol. 15, no. 5-6, pp. 275-279, 2012.

[6] V.L. Popov, "Method of reduction of dimensionality in contact and friction mechanics: A linkage between micro and macro scales," *Friction*, vol. 1, no. 1, pp. 41-62, 2013.

[7] K. Houanoh, H.P. Yin, and Q.C. He, "A simple numerical approach for solving the frictionless contact problem of elastic wavy surfaces," *Meccanica*, vol. 51, no. 2, pp. 463-473, 2016.

[8] I.G. Goryacheva, "The periodic contact problem for an elastic half-space," *Journal of Applied Mathematics and Mechanics*, vol. 62, no. 6, pp. 959-966, 1998.

[9] A. Yakovenko and I. Goryacheva, "The discrete contact problem for a two-level system of indenters," *Continuum Mechanics and Thermodynamics*, 2022.

[10] A.A. Yakovenko, "Indentation of a cylinder with different shape of the base into viscoelastic half-space," *Russian Journal of Biomechanics*, vol. 24, no. 3, pp. 294-311, 2020.

[11] I.G. Goryacheva, "Mechanics of discrete contact," *Tribology International*, vol. 39, no. 5, pp. 381-386, 2006.

[12] I.G. Goryacheva and Yu.Yu. Makhovskaya, "Discrete Contact Mechanics with Applications in Tribology," *Elsevier*, Amsterdam, 2023.

[13] L.A. Galin, "Contact Problems: The legacy of L.A. Galin," *Springer*, Dordrecht, 2008.

[14] I.G. Goryacheva and A.A. Yakovenko, "Stress state of an elastic half-space under conditions of the discrete contact," *Proceedings of the Steklov Institute of Mathematics*, 2023 (in print).