

# A Comparison Study Between the Performance Characteristics of Circular and Rectangular Recess Pads of Hydrostatic Bearings

Badr H. Bedairi<sup>a</sup> , Sultan I. Alqash<sup>a</sup> , Badr S. Azzam<sup>a,\*</sup> 

<sup>a</sup>Department of Mechanical Engineering, College of Engineering, Taibah University, Madinah, Saudi Arabia.

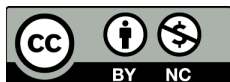
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## \* Corresponding author:

Badr Azzam  
E-mail: [bazzam@taibahu.edu.sa](mailto:bazzam@taibahu.edu.sa)

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## ABSTRACT

As known in mechanical engineering, any rotating shafts must be supported on bearings to reduce friction and wear and so decreasing the power losses and increasing the shaft life. These bearing may be rolling or sliding. Although the rolling bearings have low friction than sliding, but the sliding (journal) bearing can possess high working life than rolling ones if they furnished with good lubrication systems. These lubrication systems are divided into two types. The first type is the hydrodynamic lubrication which generates the oil pressure by the effect of squeeze action. Whereas the other type is the hydrostatic lubrication which generates the oil pressure by the effect of an external pressure source (pump).

In the hydrostatic bearing, the oil is pressurized from the external pressure source into a pocket (or pockets) manufactured in the bearing inside the surface between the shaft and the bearing to generate a projected area for carrying the radial applied load. These pockets are generally of circular shape in most applications.

In this study, a rectangular recess shape is proposed to be as the pocket instead of the circular one. The bearing characteristic performance parameters as: pressure distribution in the bearing land, the load capacity, the friction torque, the power losses, and the temperature rise are considered.

A comparison between the performance of both the circular and rectangular recess pads is given based on unifying the recess shape parameter.

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## 1. INTRODUCTION

Hydrostatic bearings are a type of bearings that operated by an external pressure source (oil pump). This is unlike the principle of operation

of hydrodynamic bearings which relies on the hydrodynamic effect between the shaft and bearing (pressure is generated in oil due to the squeeze action). These types of bearings have the advantage of low friction at low speeds of

shafts, high damping and stiffness coefficients in the normal direction of the motion with very little of adhesive wear. Hydrostatic bearings are generally composed of two main system: the first system is the pad (hydrostatic bearing itself) system, and the second system is the hydraulic system. Some hydrostatic bearings have single cavity and others have several cavities in case of multi-recess pad types, depending on the bearing load. The principle of operation of these bearings depends on pressurizing oil into the cavity found in the pad or pads through an orifice put in the inlet port. The components of the hydraulic circuit used in the hydrostatic bearings are chosen based on the maximum oil pressures resulted from the applied lifting load. The oil pressure profile is constant through the cavity area and then is reduced gradually (either linear as in the rectangular recesses and non-linear in the circular recesses) through the bearing land. At the exit of the bearing, the pressure tends to be atmospheric [1]. The thickness of the oil film generated in the bearing pad depends mainly on both the recess pressure and the applied load. Even with no relative sliding velocity between the shaft and bearing, the lubrication film can also be achieved due to the effect of the oil pressure.

Many scholars have studied these bearings numerically to solve the lubrication theory of oil motion through the bearing cavity. Others have studied them theoretically to predict the pressure distribution in the cavity for supporting the load capacity of the bearings [2-6]. Throughout those studies, the bearing performance parameters could be optimized. The studies presented in that references are depending mainly on the quality of the FEA package used, where the fluid domain of the bearing must be accurately built to describe the flow pattern inside bearing.

In the last decade, due to the improvement of computing power, a large number of researchers have used computational fluid dynamics (CFD) to analyze the static and dynamic characteristics of hydrostatic bearings. Shao et al. [7,8] have investigated the pressure distribution of hydrostatic thrust bearings by CFD, and analyzed the influence of recess dimensions, shaft rotating speed and bearing weight on the pressure field. They

carried out an experimental work to see and verify the dynamic pressure effect of recess depth on the performance of the hydrostatic-thrust bearings.

Osman et al. [9] introduced kinetic load to study the dynamic characteristics of ring thrust bearing and proposed an optimal flow rate to achieve a better supporting performance.

Salem and et al. [10] presented an optimum design of hydrostatic journal bearings. They could minimize the power losses in that type of bearings. In their study, they applied the Rosenbrock's optimization [11] method to get the optimum parameters of the bearing which minimized the power losses in bearing. They concluded that to minimize the power losses in the hydrostatic bearings, the clearance between the bearing and shaft must be minimized and the high pressure supplied to bearing must be avoided with recommendation of oils have high viscosity.

Wang et al. [12] presented in their paper a theoretical study using a CFD method as well as 3D meshing for the hydrostatic bearings based on an unstructured meshing scheme and turbulence model. They accompanied their study with an experimental work to verify the theoretical results.

By comparing these results to the traditional published empirical formulae, it was found that the 3D meshing and CFD analysis is more accurate and can adapt to the analysis of various shapes of hydrostatic bearings.

Throughout the above references presented on that type of bearings, one can conclude that the performance characteristics of these hydrostatic bearings are depending mainly on various parameters. Some of these parameters are related to bearing dimensions (bearing diameter, length and clearance between bearing and shaft). Other parameters are related to the carried shaft (load and speed). In addition to parameters, which related to recess geometry (shape and dimensions) and the oil properties as the dynamic viscosity of oil.

In this paperwork, two types of pads having different shapes, the first has a circular recess and the other one has a rectangular recess are studied to see the influence of these parameters on their performance.

## 2. ANALYRICAL STUDY

The pressure distribution through the bearing land could be obtained from the Reynolds equation [1] as follows: -

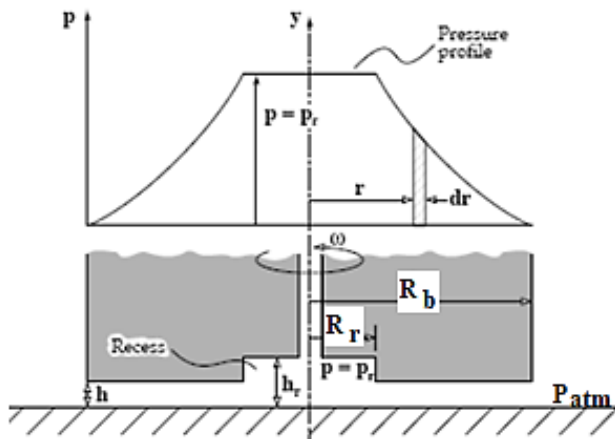
$$q_x = -\frac{h^3}{12\mu} \frac{\partial p}{\partial x} \quad (1)$$

By considering, the oil flow from the land between the bearing and shaft is in one direction (in the circular pad through the radial direction only and in the rectangular pad in the opposite direction of the pad length). Therefore, equation (1) can be rewritten as follows:

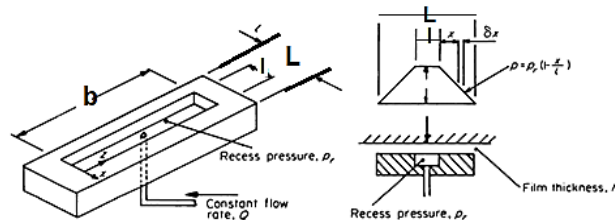
$$q_x = -\frac{h^3}{12\mu} \frac{dp}{dx} \quad (2)$$

Where,

$q_x$  = is the flow rate per unit width =  $Q/2\pi r$  for circular pad ( $r$  = pad radius, Fig. (1a)) =  $Q/2b$  for rectangular pad ( $b$  = pad length, Fig. (1b) [13].



(a) Circular recess bearing



(b) Rectangular recess bearing

**Fig. 1.** Flow rate through the bearing pad.

By integrating Eqn. (2) and applying the boundary conditions, one can get the flow rate and the pressure distribution through the bearing clearance as given in the following sections. By substituting  $Q$  in those equations, the

pressure distribution is expressed in terms of the pressure and bearing geometry as shown in the following sections.

### 2.1 Pressure distribution

The oil enters the cavity of the bearing with a high-pressure  $P_r$  and then escapes through the land between the bearing and shaft. Therefore, the oil will leak with a decrease variation from  $P_r$  at the recess to  $P_{atm}$  at the exit. That variation is non-linear in the circular pads and linear in the rectangular ones, as shown in the following subsections.

#### a. Circular recess pad pressure distribution

The following equation gives the pressure distribution in the circular recess, as shown in figure (1a) [12]:

$$P = P_r \frac{\ln\left(\frac{R}{r}\right)}{\ln\left(\frac{R_b}{R_r}\right)} \quad (3)$$

Where;

$R_b$  = bearing radius

$R_r$  = recess radius

$r$  is a general position in the bearing land  $R_r \leq r \leq R_b$

By putting  $\bar{p} = \frac{p}{p_r}$  and  $\bar{r} = \frac{r}{R_b}$ , one can re-write equation (3) as follows:

$$\bar{p} = -\frac{1}{\ln\left(\frac{R_b}{R_r}\right)} \ln \bar{r} \quad (4)$$

#### b. Rectangular pad pressure distribution

Similarly, as above for the circular recess pads, equation (5) gives the pressure distribution in the rectangular recess at a general position  $x$ , as shown in figure (2):

$$P = P_r \left( \frac{L-2x}{L-l} \right) \quad (5)$$

By putting  $\bar{p} = \frac{p}{p_r}$  and  $\bar{r} = \frac{x}{L}$ , one can re-write equation (5) as follows:

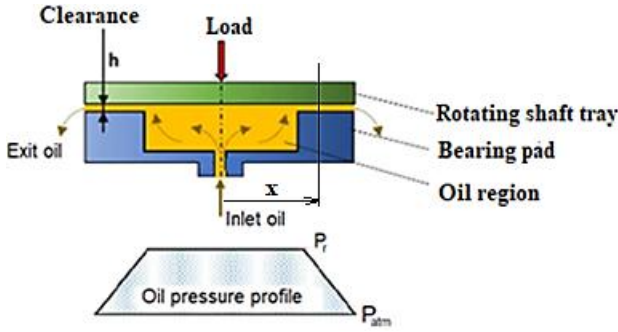
$$\bar{p} = \frac{L}{L-l} [1 - 2r'] \quad (6)$$

Where,

$L$  = bearing pad width

$l$  = bearing recess width

$x$  is a distance at a general position in the bearing land;  $l/2 \leq x \leq L/2$



**Fig. 2.** Oil pressure distribution through the bearing pad with rectangular recess

Hint, the oil leakage through the bearing of this type is assumed to flow through the bearing width sides while, its leakage through the bearing length sides is ignored.

## 2.2 Flow rate

The flow rate, which enters the bearing recess could be obtained through differentiating the Reynold equation. Rearranging the parameters of that equation to be in simple forms, as given in equations (7 and 8), for the circular and rectangular recesses, respectively. This amount oil flow rate, which needed to be pushed from the pump to maintain the film thickness of oil as  $h$  can be obtained as in the following sections.

### a. Flow Rate in Circular Recess Pads

The following equation gives the oil flow rate in the circular recess (after differentiating the Reynold equation): -

$$Q = \frac{\pi h^3 P_r}{6\mu} \frac{1}{\ln\left(\frac{R_b}{R_r}\right)} \quad (7)$$

Where;

$h$  is oil thickness in the bearing land (= clearance between shaft and bearing) as shown in figure (2).

$\mu$  is the oil dynamic viscosity

### b. Flow Rate in Rectangular Recess Pads

The flow rate in the rectangular recess pads can be obtained similarly as for the circular recess pads. Equation (8) gives the oil flow rate in the rectangular recess.

$$Q = \frac{h^3 p_r}{3\mu} \left( \frac{b}{L-l} \right) \quad (8)$$

## Non-dimensional flow parameter

Equations (7) & (8) of flow capacity can be rewritten in a non-dimensional form for flow parameter ( $\bar{Q}$ ) for the circular and rectangular shape parameter ( $\beta$ ) as follows in equation (9).

$$\bar{Q} = Q / \frac{h^3 P_r}{\mu} = \beta \quad (9)$$

Where,  $\bar{B}$  is a non-dimensional shape parameter

$$= \frac{\pi}{6} \frac{1}{\ln\left(\frac{R_b}{R_r}\right)} \quad \text{for circular recess pads}$$

$$= \frac{1}{3} \left( \frac{b}{L-l} \right) \quad \text{for rectangular recess pads.}$$

The variation of the flow rate parameter ( $\bar{Q}$ ) with the non-dimensional shape parameter  $\beta$  is shown in figure (3) for both the circular recess and rectangular recess pads.

## 2.3 Load capacity

The total load which can be supported by the bearing is obtained by multiplying the pressure and area. This area is divided into two regions: the recess area and the bearing land area. The pressure in the recess area is constant ( $P_r$ ), so the load is simply obtained. On the other hand, the pressure in the land is varying, so to obtain the load carried by the oil in this land, one must make an integration for the pressure load over a specific area of the bearing land to get the load carried by the oil in the bearing land. The total load capacity could be obtained for the circular recesses and rectangular recesses as given in equations (10 and 11), respectively.

### a. Load capacity of bearings with circular recess pads

$$W = \frac{\pi P_r}{2} \left[ \frac{R_b^2 - R_r^2}{\ln\left(\frac{R_b}{R_r}\right)} \right] \quad (10)$$

### b. Load capacity of bearings with rectangular recess pads

$$W = \frac{P_r b}{2} (L + l) \quad (11)$$

Where,  $b$  is the recess length, refer to figure (1b).

### Non-dimensional load parameter

Similarly, as in the flow rate calculation, equations (10) & (11) of load capacity can be re-written in a non-dimensional load parameter ( $\bar{w}$ ) in terms of the non-dimensional shape parameter ( $\bar{\beta}$ ) as follows in equation (12).

$$\begin{aligned}\bar{w} &= W/p_r R_b^2 = 3(1 - \alpha^2)\beta \text{ for circular recess pads} \\ &= W/p_r L^2 = \frac{3}{2}(1 - \alpha^2)\beta \text{ for rectangular recess pads}\end{aligned}\quad (12)$$

$$\begin{aligned}\text{Where, } \alpha &= \frac{R_r}{R_b} \text{ for circular recess pads} \\ &= \frac{l}{L} \text{ for rectangular recess pads.}\end{aligned}$$

The variations of the load parameter ( $\bar{w}$ ) with the non-dimensional shape parameter  $\beta$  and for different values of ( $\alpha$ ) are shown in figures (4 and 5) for both the circular recess and rectangular recess pads, respectively.

### 2.4 Friction torque

The friction resistance of a rotating hydrostatic bearing with circular recesses or rectangular ones depends mainly on the shear stress resulted due to the applied load, the oil viscosity, the speed of the shaft and the dimensions of the pad. The friction total torque ( $T_f$ ) can be found for both the circular and rectangular recesses, as follows [10]: The total friction torque has two components: one related to the recess area and the other to the bearing land area.

$$T_f = T_r + T_l$$

Where;  
 $T_r$  is the friction torque at the recess and  $T_l$  is the friction torque in the bearing land

$$\begin{aligned}T_f &= \int_A r dF \\ dF &= \tau dA\end{aligned}$$

The shear stress ( $\tau$ ) can be found as:

$$\tau = \mu \frac{du}{dz} = \mu \frac{u}{h}$$

Where,  $u$  is the linear speed of the shaft,  $\mu$  is the oil viscosity and  $h$  is the oil film thickness

### Friction torque in circular and rectangular pads bearings

The differential form of the friction form,  $dF$ , has two components: one for the recess and the other for the land in the bearing, as follows:

$$dF = \mu \frac{U}{h_r} r d\theta dr + \mu \frac{U}{h} r d\theta dr \quad (13)$$

Substituting for  $U = 2\pi r n$  with considering constant viscosity and velocity and perform an integration for equation (13) yields the friction torque ( $T_f$ ) for the circular recess pads and rectangular recess pads as follows in equations (14 and 15), respectively:

$$T_f = \frac{\pi^2 \mu n}{h_r} R_r^4 + \frac{\pi^2 \mu n}{h} (R_b^4 - R_r^4) \quad (14)$$

$$T_f = \frac{2\pi n \mu}{h_r} (lb)^2 + \frac{4\pi n \mu L b}{h} (L - l)b \quad (15)$$

It is shown that the first term in the above equations is very small relative to the second term where the oil thickness in the pad recess (within mm-scale) is much more than the oil thickness in the bearing land (within  $\mu$ m-scale). Therefore, the first term in that equations can be neglected relative to the second term which means that the torque in the bearing land can be considered only in the bearing land as follows:

$$T_f = \frac{\pi^2 \mu n}{h} (R_b^4 - R_r^4) \quad (16)$$

$$T_f = \frac{4\pi n \mu L b}{h} (L - l)b \quad (17)$$

### 2.5 Power Loss

The power loss from the bearing is mainly due to the frictional torque. This power loss will be transferred to heat generated in the oil. This heat generation in the oil will raise its temperature before transferred to ambient through the operating surfaces of bearing.

The power loss in both the circular and rectangular pads will be as follows in equations (18 & 19), respectively:

$$H_g = \frac{2\pi^3 \mu n^2}{h} (R_b^4 - R_r^4) \quad (18)$$

$$H_g = \frac{8\pi^2 \mu n^2}{h} (lb)(L - l)b \quad (19)$$

## 2.6 Temperature rise of oil

The heat generated in the oil will raise its temperature by a temperature rise  $\Delta T$ . This temperature rise can be calculated from the following equation as follows in equation (20):

$$\Delta T = \frac{Hg}{\rho C_v Q} \quad (20)$$

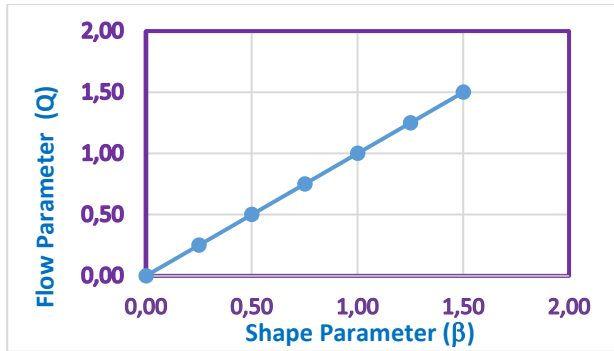
Where,

$Q$  is the lubricant flow rate (from equations 7 and 8)

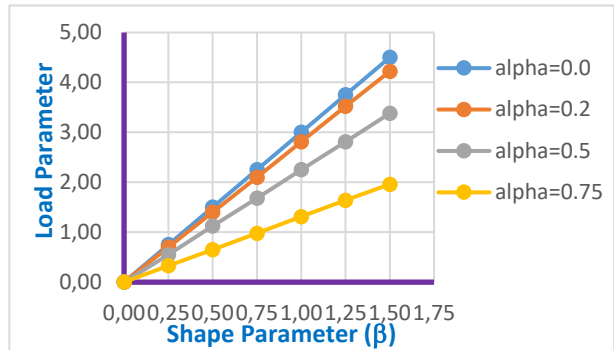
$\rho$  is the density of the lubricant

$C_v$  is the specific heat of the lubricant

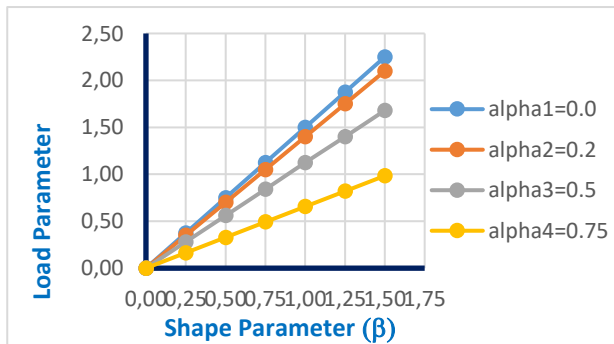
$\Delta T$  is the oil temperature rise



**Fig. 3.** Variation of Non-dimensional flow parameter ( $\bar{Q}$ ) with shape parameter ( $\beta$ ) for both circular and rectangular recess pads.



**Fig. 4.** Variation of Non-dimensional Load parameter ( $\bar{w}$ ) with shape parameter ( $\beta$ ) for different  $\alpha_r$  ratios for rectangular recess pads.



**Fig. 5.** Variation of Non-dimensional Load parameter ( $\bar{w}$ ) with shape parameter ( $\beta$ ) for different  $\alpha_c$  ratios for rectangular recess pads.

## 3. CASE STUDY

In this case study, two hydrostatic bearings; the first with a circular recess pad and the second with a rectangular recess pad, supporting a shaft rotates with a speed  $100 \text{ RPM}$  and supporting a load of  $10 \text{ tons}$ . The bearing recess depth ( $h_r$ ) is taken as  $5 \text{ mm}$  and the land clearance ( $h$ ) as  $100 \mu\text{m}$  for both types of pads. SAE-30 oil with a viscosity =  $75 \text{ cp}$  ( $\mu = 75 \text{ mPa.s}$ ). The bearing characteristics and specifications are as follows:

### Circular pad bearing

- Circular pad dimensions ( $R_b, R_r$ ) =  $200, 50$ , respectively

### Rectangular pad bearing

- Recess pad dimensions ( $L, b, l$ ) =  $300, 195, 125$  mm, respectively.

The recess dimensions of the bearing pads have been taken with an equal pad surface area and an equal non-dimensional shape parameter ( $\beta = 0.377$ ) for both types of bearings to support the same load for the comparison study of the results.

The distribution of pressure in the bearing land for the two types of bearing pads is shown in figure (1). The pressure distribution of the oil in the bearing land is linear for the rectangular pads and non-linear for the circular pads.

## 4. RESULTS AND DISCUSSIONS

By substitution in equations (10) and (11), and by knowing the applied load as  $10 \text{ tons}$ , the required inlet pressure is obtained as  $23.54$  and  $24.14 \text{ bars}$  for the circular recess pad and rectangular recess pad, respectively. This shows that the bearings of rectangular pads need an inlet oil pressure more than that of the bearings of circular ones for supporting the same supporting load.

The oil flow rate entering the bearing pad is obtained from equation (9) as  $0.712 \text{ l/min}$  for the two types of the bearing pads which means that the bearing pad configuration has no effect on the inlet flow rate. On other hand, the required power to operate the oil pump used in the lubrication system in the rectangular recess pads will be higher than that of the circular recess pads as a result of the higher pressure in the rectangular pads than that of the circular ones.

The friction torque in both types of pads are obtained from equations (16 and 17) as 19.63 and 31.43 Nm for the circular and rectangular recess pads, respectively.

The power loss through the two types of pads is obtained from equations (18 and 19) as 205 and 328 watts for the circular and rectangular recess pads, respectively. Regarding the temperature rise in the lubricating oil (specific heat = 1675 J/kg °C and specific weight = 0.85), it is determined from equation (20) as 12.13 and 19.41 °C for the circular recess and rectangular recess pads, respectively.

## 5. CONCLUSIONS

From the analytical studies performed in this work for two types of pads having different recess configurations (circular and rectangular) and supporting similar load (shaft force and speed) with similar oil properties and same non-dimensional scale parameter, the following conclusions can be withdrawn:

1. Increasing the shape parameter ( $\beta$ ) increases the flow rate parameter ( $Q$ ) in both types of pads.
2. The load capacity parameter ( $\bar{w}$ ) of the bearing increases with increasing the shape parameter ( $\beta$ ) for both types of pads having different values of the size ratio parameter ( $\alpha$ ).
3. The flow rate in both types of recesses is same for both configurations of pads.
4. The bearings with rectangular recess pads need an inlet oil pressure higher than that of the circular pads by about 2.5%.
5. The power losses (heat generation in the lubricating oil) due to friction in the rectangular recess pads is higher than that of the circular ones by about 12.5%.
6. The temperature rise in the rectangular recess pads is higher than that of the circular ones by about 37.5%.
7. Due to the sharp corners of the rectangular recess pads, there will be a stress concentration which may affect the stresses resulted in the bearing pad due to the inside high pressure of the lubricating oil in the recess and so reduces its safety factor. While the circular recess pad gets rid of that effect.

8. As a final conclusion, the performance characteristics of bearings having circular recess pads are better than that of the rectangular recess pads in every respect (oil pressure, power losses, temperature rise and stress concentration).

## REFERENCES

- [1] J. Halling, "Principles of tribology", Red Globe Press London, 1978.
- [2] Z.S. Liu, G. Zhang, and H.J. Xu, "Performance analysis of rotating externally pressurized air bearings", *Journal of Process Inst. Mech. Eng. Part J. Eng. Tribology*, vol. 223, pp. 653–663, 2009.
- [3] C.B. Khatri and S.C. Sharma, "Influence of textured surface on the performance of non-recessed hybrid journal bearing operating with non-Newtonian lubricant", *Int. Journal of Tribology*, vol. 95, pp. 221–235, 2016.
- [4] G.J. Zhang, J. Li, Z.X. Tian, Y. Huang, and R.C. Chen, "Film Shape Optimization for Two-Dimensional Rough Slider Bearings", *Int. Journal of Tribology*, vol. 59, pp. 17–27, 2015.
- [5] F. Shen, C.L. Chen, and Z.M. Liu, "Effect of Pocket Geometry on the Performance of a Circular Thrust Pad Hydrostatic Bearing in Machine Tools", *Journal of Tribology*, Trans. vol. 57, pp. 700–714, 2014.
- [6] P. Liang, C.H. Lu, W. Pan, and S.Y. Li, "A new method for calculating the static performance of hydrostatic journal bearing", *Journal of Tribology*, vol. 77, pp. 72–77, 2014.
- [7] J.P. Shao, L.M. Zhou, H.M. Li, X.D. Yang, Y.Q. Zhang, and M.S. Chi, "Influence of the oil Cavity Depth on Dynamic Pressure Effect of Hydrostatic Thrust Bearing", in *Proceedings of the 2009 International Conference on Intelligent Human-Machine Systems and Cybernetics*, Hangzhou, China, 26–27 August 2009, pp. 11–14.
- [8] J.P. Shao, G.D. Liu, and X.D. Yu, "Simulation and experiment on pressure field characteristics of hydrostatic hydrodynamic hybrid thrust bearings", *Ind. Int. Journal of Lubrication and Tribology*, vol. 71, pp. 102–108, 2018.
- [9] T.A. Osman, Z. Safar, and M.O.A. Mokhtar, "Design of annular recess hydrostatic thrust bearing under dynamic loading", *Tribology Int. Journal*, vol. 24, pp. 137–141, 1991.



- [10] F. Salem, M. El-Sherbiny, and N. El-Hefnawy, "Optimum design of hydrostatic journal bearings, Part II: Minimum power losses", *Journal of Engineering and Applied Sciences*, vol. 2, pp. 171-184, 1983.
- [11] H.H. Rosenbrock, "An automatic method for finding the greatest or least value of a function", *Computer Journal*, vol. 3, pp. 175-184, 1960.
- [12] Y. Wang, H. Wu, and Y. Rong, "Analysis of hydrostatic bearings based on a unstructured meshing scheme and turbulence model", *Machine*, 2022.
- [13] W. Shizhu and H. Ping, "Principle of Tribology", John Wiley & Sons (Asia), Tsinghua University Press, 2012.

## NOMENCLATURE

$p$  = Bearing land pressure  
 $p_r$  = Recess pressure  
 $h$  = Lubricant film thickness in the bearing land  
 $h_r$  = Lubricant thickness in recess (recess depth)  
 $\mu$  = Lubricant dynamic viscosity  
 $R_b$  = Outer radius of the bearing  
 $R_r$  = Radius of the circular recess  
 $a, b$  = Side lengths of the rectangular recess  
 $q_x$  = Flow rate per unit width  
 $Q$  = Lubricant flow rate  
 $n$  = Speed of the shaft  
 $\rho$  = Density of the lubricant  
 $C_v$  = Specific heat of the lubricant  
 $\Delta T$  = Temperature rise in oil  
 $\beta$  = Non-dimensional shape parameter  
 $\alpha$  = Non-dimensional size parameter